

Team Number:	apmcm24212499
Problem Chosen:	C

2026 APMCM summary sheet

For Problem 1, this study uses the ARIMAX model to analyze the development of China's pet industry over the next three years, incorporating three economic indicators: disposable income, consumer confidence index, and consumer price index. These factors explain the drivers of the pet industry from the perspectives of income, psychological expectations, and prices, thereby improving the model's explanatory power and prediction accuracy. Based on the results, the number of cats is expected to reach **72.07 million** in 2024 and **86.64 million** in 2026, while the number of dogs is projected to decline from **51.88 million** in 2024 to **49.77 million** in 2027.

For Problem 2, first, the ARIMAX model is applied to predict the development of the pet industry in the United States, France, and Germany. The predicted number of cats and dogs in the United States for 2024 are **71.66 million** and **77.17 million**, respectively, while the corresponding figures for France are **19.85 million** and **12.94 million**. Using the historical proportion of these four countries' pet populations and exponential smoothing, the global trend and specific values for future pet populations were predicted. The results show that the global population of pet cats and dogs in 2025 will be **448.97 million** and **499.81 million**, respectively. Furthermore, the model predicts that by around **2027**, the number of cats globally will surpass the number of dogs.

For Problem 3, this study employs the Holt linear trend model, considering that pet food production and exports are influenced by various factors, resulting in significant fluctuations. To account for this, a **90% confidence interval** is used. The results show that the 90% confidence interval for production in 2024 is estimated to be between **96.99 billion RMB** and **335.60 billion RMB**, while the export value is projected to range from **22.97 billion RMB** to **54.44 billion RMB** within the same confidence interval.

For Problem 4, based on international trade theory, this study constructs an econometric model to quantitatively analyze the relationship between tariff rates and export growth rates. The analysis calculates the change in exports under tariff rates ranging from **5% to 50%**. The results indicate that at a tariff rate of **10%**, the export growth rate is **-6.80%**; at **30%**, it is **-32.20%**; and at **50%**, it is **-53.60%**.

Keywords: ARIMAX Model Exponential Smoothing Holt Linear Trend Model
Econometric Model

Contents

1. Problem Restatement	1
1.1 Analysis and Forecast of Domestic Pet Industry Development.	1
1.2 Global Pet Industry Development and Future Demand Forecast	1
1.3 Development and Export Forecast of China's Pet Food Industry	1
1.4 Impact of Economic Policies on China's Pet Food Industry and Response Strategies	2
2. Model Hypotheses	2
3. Notation Explanation	2
4. Country's Future Trend Forecast Based on the ARIMAX Model	3
4.1 Data Preparation	3
4.2 ARIMAX Model	4
4.3 Forecasting Chinese Pet Industry Using the ARIMAX Model	6
4.4 Forecasting the U.S. Pet Industry Using the ARIMAX Model	7
4.5 Forecasting French Pet Industry Using the ARIMAX Model	9
4.6 Forecasting German Pet Industry Using the ARIMAX Model	11
5. The World's Pet Industry Forecast Based on Exponential Smoothing	14
5.1 Exponential Smoothing Model	14
5.2 Model Results	14
6. Forecasting China's Pet Food Production and Export Using Holt Model ...	16
6.1 Current Analysis	16
6.2 The Holt Model	16
6.3 Model Results	17
7. Econometric Model on Tariff Impact on China's Pet Food Industry	19
7.1 Econometric Model	19
7.2 Model Results	19
7.3 Sustainable Development Strategies	20
8. Further Discussion on the Model	21
8.1 1. Model Advantages	21
8.2 2. Model Limitations	21
9. References	22
10. Appendix	22

I. Problem Restatement

1.1 Analysis and Forecast of Domestic Pet Industry Development

Over the past five years, China's pet industry has grown rapidly, with significant differences in growth rates and market changes among different pet types (e.g., cats and dogs). Based on the data provided in Appendix 1 and additional data collected by the team, it is necessary to analyze the growth trends of China's pet industry and identify the key factors influencing the industry's development. By constructing appropriate mathematical models (e.g., time series models), the development of China's pet industry over the next three years can be predicted, providing data-driven support for industry decision-making.

1.2 Global Pet Industry Development and Future Demand Forecast

In recent years, the pet industry in regions such as Europe and the United States has experienced remarkable growth, particularly in the demand for pet food. Based on the data provided in Appendix 2 and additional data collected, it is necessary to analyze the development of the global pet industry from the perspective of pet types. Using appropriate mathematical models (e.g., demand forecasting models or regression models), the global demand for pet food over the next three years can be forecasted, offering insights for global market planning and production strategies.

1.3 Development and Export Forecast of China's Pet Food Industry

China, as a crucial part of the global pet food supply chain, has continuously expanded its pet food production and export scale in recent years. By combining the data in Appendix 3 regarding China's pet food production and export with additional data, it is necessary to analyze the growth trends of the industry and the main drivers of domestic and international markets. Based on forecasting models, estimates of China's pet food production and export over the next three years can be provided, offering forward-looking recommendations for industry stakeholders and policymakers.

1.4 Impact of Economic Policies on China's Pet Food Industry and Response Strategies

With the introduction of new economic policies in Europe and the United States (e.g., tariff adjustments), China's pet food export industry is inevitably affected. It is necessary to construct suitable mathematical models, leveraging data from the appendices, additional data collected by the team, and results from previous questions, to quantify the impact of economic policies. Based on the results of the analysis, sustainable development strategies should be formulated to help China's pet food industry adapt to policy changes and ensure stable growth in a complex international environment.

II. Model Hypotheses

1. The ARIMAX model assumes that the time series of the target variable is stationary. If the original series is non-stationary, it needs to be made stationary by differencing or other transformations.
2. The model residuals should be independently and identically distributed, meaning they are uncorrelated and follow a normal distribution. If the residuals show correlation, it suggests that some variables might be missing or the model is underfitted.
3. Exponential smoothing assumes that changes in pet numbers follow a smoothed trend, and future changes are mainly determined by the existing trend. If sudden events occur, the model may not adapt well.
4. The data is assumed to follow a linear trend over time, which the Holt model captures using additive or multiplicative forms. If the data trend is nonlinear, the Holt model might fail to fit accurately.

III. Notation Explanation

Symbol	Description
y_t	Target variable at time t
c	Constant term, the intercept in the model
ϕ_i	Coefficients of the autoregressive terms
θ_j	Coefficients of the moving average terms

Symbol	Description
p	Order of the autoregressive terms
q	Order of the moving average terms
ϵ_t	White noise
$X_{t,k}$	Exogenous variables
μ_k	Mean of the exogenous variable
σ_k	Standard deviation of the exogenous variable
β_k	Regression coefficients of the exogenous variable
S_t	Smoothed value for the current year
b_t	Trend value for the current year
Y_t	Actual value for the current year
T_t	Trend value over time t
L_t	Level value over time t
$t_{(\alpha/2, n-2)}$	Critical value of the t -distribution
α	Smoothing coefficient
β	Smoothing coefficient
Q_d	Import demand quantity
P	Price of the product
t	Tariff rate
ω	Price elasticity

IV. Country's Future Trend Forecast Based on the ARIMAX Model

4.1 Data Preparation

(1) Disposable Income

Disposable income reflects residents' economic capacity and is a direct indicator of consumption ability. The pet industry, as a non-essential consumption sector, heavily relies on the increase in per capita disposable income. An increase in disposable income encourages spending on pet food, medical care, supplies, and other premium services. It directly represents residents' economic strength and serves as a fundamental driving force in the pet industry.

Year	2019	2020	2021	2022	2023
Disposable Income (CNY)	30,732.8	32,188.8	35,128.1	36,883.3	39,218

(2) Consumer Confidence Index

The Consumer Confidence Index (CCI) reflects residents' expectations of the future economy and their consumption tendencies. As an emotional indicator, it directly influences willingness to spend on non-essential items. High CCI values indicate greater consumer willingness to make high-value purchases, such as pet supplies and veterinary services. Conversely, a lower CCI may hinder growth in the pet industry, particularly in the premium segment. The CCI serves as a leading indicator for gauging changes in potential demand.

Year	2019	2020	2021	2022	2023
CCI	124.6	119.5	111.1	95	88.8

(3) Consumer Price Index

The Consumer Price Index (CPI) reflects changes in the overall price level, directly impacting residents' purchasing power and consumption habits. A high CPI indicates decreased purchasing power, potentially negatively affecting pet industry consumption, especially for price-sensitive items like pet food and supplies. The CPI provides insights into consumption decisions from a macroeconomic perspective.

Year	2019	2020	2021	2022	2023
CPI (%)	2.9	2.5	0.9	2.0	0.2

4.2 ARIMAX Model

(1) Data Processing

An important prerequisite for time series analysis is stationarity, where statistical properties (e.g., mean, variance, autocorrelation) remain constant over time. Many economic datasets are non-stationary due to long-term trends or seasonal variations. To achieve stationarity, this study applied differencing, mathematically expressed as:

$$y'_t = y_t - y_{t-1}$$

After differencing, the stationarity of the series was validated using the Augmented Dickey-Fuller (ADF) test. If the data remained non-stationary, further differencing or other transformation techniques were applied.

(2) Model Foundation

The ARIMAX model is an extension of the ARIMA model, with its basic form expressed as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q}$$

(3) Inclusion of Exogenous Variables

To enhance forecasting performance, exogenous variables were introduced to the ARIMA model, forming the ARIMAX model. Exogenous variables are factors that explain changes in the target variable but are independent of it. In this study, the exogenous variables include disposable income ($X_{t,1}$), the Consumer Confidence Index ($X_{t,2}$), and the Consumer Price Index ($X_{t,3}$).

To eliminate the effects of differences in units and magnitudes, exogenous variables were standardized as follows:

$$X'_{t,k} = \frac{X_{t,k} - \mu_k}{\sigma_k}$$

The extended ARIMAX model is expressed as:

$$y_t = c + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{k=1}^m \beta_k X_{t,k} + \epsilon_t$$

By incorporating exogenous variables, the ARIMAX model captures both the time series characteristics of the target variable and the external driving effects, improving forecasting accuracy.

(4) Model Evaluation

Model performance was evaluated by comparing actual values to fitted values, calculated as:

$$\hat{y}_t = c + \phi_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \sum_{k=1}^3 \beta_k X_{t+h,k}$$

Residual analysis assessed model fitting quality. Ideally, residuals should resemble random white noise, with a mean close to zero and no significant autocorrelation.

4.3 Forecasting Chinese Pet Industry Using the ARIMAX Model

Using the above model and data, we forecast the development of the Chinese pet industry, starting with the forecast difference plot and forecast fitting plot.

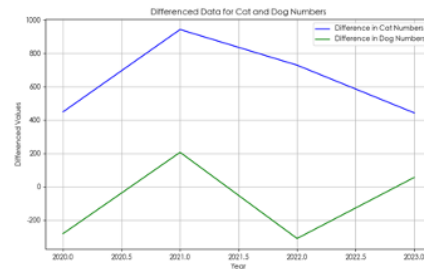


Figure 1 Forecast Difference Plot for the Chinese Pet Industry

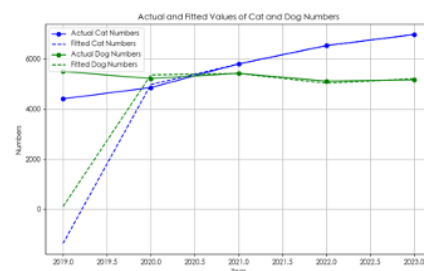


Figure 2 Forecast Fitting Plot for the Chinese Pet Industry

From the figures, we can observe that the difference values fluctuate over time, providing valuable support for building the forecasting model. Additionally, the fitted values align closely with the actual values, indicating that the ARIMAX model performs well in capturing the trends and fluctuations of the time series.

Table 2 Forecast Results for the Chinese Pet Industry (in 10,000s)

Year	Cat Forecast	Dog Forecast
2024	7206.63	5187.79
2025	8025.77	5045.03
2026	8663.70	4976.54

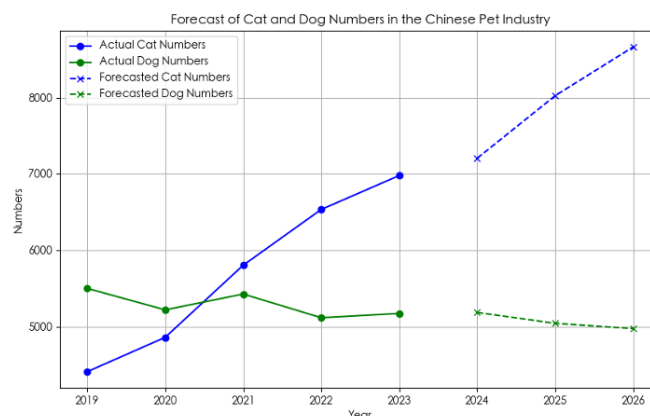


Figure 3 Forecast Plot for the Chinese Pet Industry

The number of cats is projected to grow from 72.06 million in 2024 to 86.64 million in 2026, reflecting the ongoing popularity of the pet cat market. This significant growth creates vast opportunities in the industry. High-end services for cats, such as premium cat food, medical care, and smart supplies, may become major growth areas.

Meanwhile, the number of dogs is forecasted to decrease slightly, from 51.88 million in 2024 to 49.77 million in 2026, suggesting that the dog market may be reaching saturation or experiencing constraints due to policy or environmental factors. However, this trend does not imply a contraction in the dog-related market; rather, it suggests a need for companies to reposition their market strategies. Segments like premium care, health food, and behavioral training for dogs could present future growth opportunities.

4.4 Forecasting the U.S. Pet Industry Using the ARIMAX Model

Data Selection

For forecasting the U.S. pet industry, we selected three economic indicators: disposable income, consumer confidence index, and consumer price index. These indicators cover three critical dimensions—economic capability, psychological expectations, and price levels—closely related to the development of the pet industry.

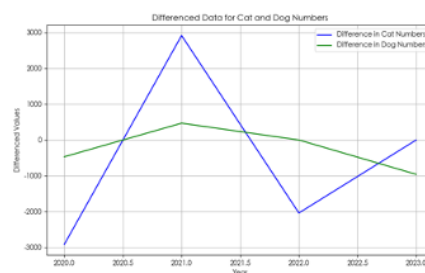
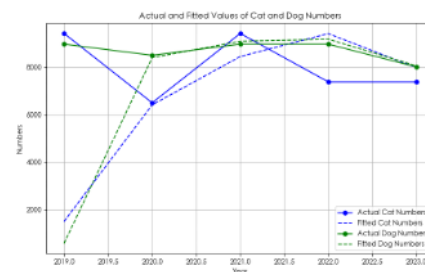
Source: U.S. Bureau of Economic Analysis (BEA)

Model Results

From the difference plot, we can see that cat numbers experienced significant growth between 2020 and 2021, with a sharp increase in the difference values, indicating a rapid growth rate. Between 2021 and 2022, the difference value fell sharply to negative,

Table 3 Relevant U.S. Economic Data

Year	Disposable Income	Consumer Confidence Index	Consumer Price Index
2019	45579	120	1.8
2020	48219	90	1.2
2021	50600	102.5	4.7
2022	52000	100	8.0
2023	53500	105	4.1

**Figure 4 Forecast Difference Plot for the U.S. Pet Industry****Figure 5 Forecast Fitting Plot for the U.S. Pet Industry**

showing stagnation or even a slight decrease in cat numbers. From 2022 to 2023, the difference value rose again, indicating high volatility. Overall, 2021 serves as a turning point for growth and decline.

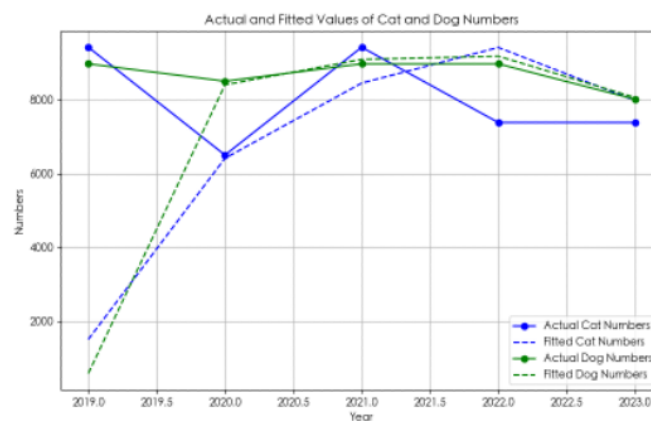
For dog numbers, the difference value remains close to zero, showing relatively stable numbers. However, starting in 2021, there is a gradual decline.

From the fitting plot, the fitted values for cats slightly exceed the actual values but are overall close. For dogs, the model fits exceptionally well, accurately predicting the stable trend.

In 2024, the number of cats is forecasted at 71.66 million, and the number of dogs

Table 4 Forecast Results for the U.S. Pet Industry (in 10,000s)

Year	Cat Forecast	Dog Forecast
2024	7165.66	7717.43
2025	7260.58	7485.29
2026	7370.57	7299.40

**Figure 6 Forecast Plot for the U.S. Pet Industry**

at 77.17 million. While dog numbers remain higher, the gap is narrowing. By 2026, cat numbers are expected to rise to 73.71 million, surpassing the 72.99 million dogs for the first time.

4.5 Forecasting French Pet Industry Using the ARIMAX Model

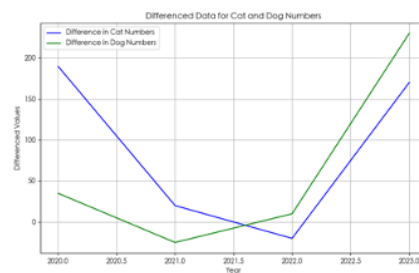
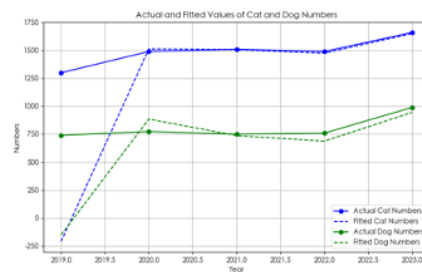
Data Selection

For forecasting the French pet industry, we selected two economic indicators: the consumer confidence index and the consumer price index. These indicators cover key dimensions of psychological expectations and price levels, which are highly relevant to the development of the pet industry. Due to data availability challenges, disposable income was not included in the analysis.

Source: National Institute of Statistics and Economic Studies (INSEE)

Table 5 Relevant French Economic Data

Year	Consumer Confidence Index	Consumer Price Index
2019	101	1.3
2020	95	0.5
2021	99	2.1
2022	82	5.9
2023	85	5.7

**Figure 7 Forecast Difference Plot for the French Pet Industry****Figure 8 Forecast Fitting Plot for the French Pet Industry**

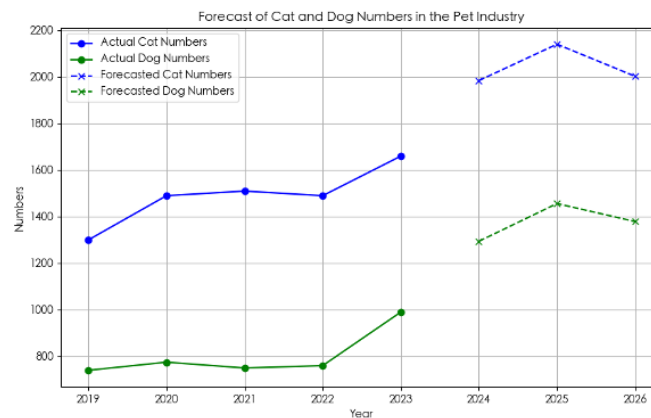
Model Results

From the difference plot, the difference values for cats reached their lowest point in 2021 but then showed a steady upward trend, with significant growth observed in 2023. This indicates an accelerating growth rate for cats. For dogs, the difference values also hit their lowest point in 2021 and gradually rebounded, peaking in 2023, suggesting a recovery in demand.

From the fitting plot, the actual values (solid blue line) and fitted values (dashed blue line) are nearly identical, showing the model's excellent performance in capturing the historical data trends for both cats and dogs.

Table 6 Forecast Results for the French Pet Industry (in 10,000s)

Year	Cat Forecast	Dog Forecast
2024	1984.56	1294.23
2025	2140.82	1456.32
2026	2003.07	1379.05

**Figure 9 Forecast Plot for the French Pet Industry**

The number of cats is expected to grow from 19.85 million in 2024 to 21.41 million in 2025, reflecting robust growth. In 2026, the number of cats may decrease slightly to 20.03 million, suggesting a mild market adjustment.

For dogs, the number is forecasted to grow from 12.94 million in 2024 to 14.56 million in 2025, before declining slightly to 13.79 million in 2026. Although the growth rate is slower than that of cats, the overall trend is more stable, with less pronounced market corrections.

4.6 Forecasting German Pet Industry Using the ARIMAX Model

Data Selection

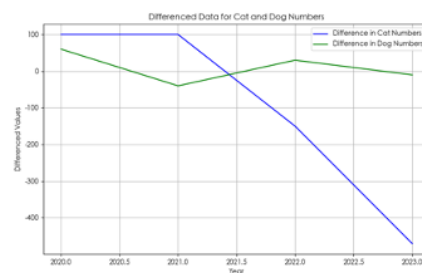
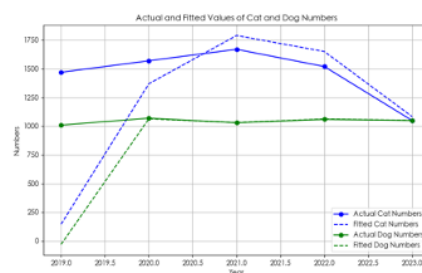
For forecasting the German pet industry, we selected three economic indicators: disposable income, consumer confidence index, and consumer price index. These indicators complement each other by explaining the driving factors of the pet industry from the perspectives of income, psychological expectations, and price levels, enhancing the model's explanatory power and predictive accuracy.

Table 7 Relevant German Economic Data

Year	Disposable Income	Consumer Confidence Index	Consumer Price Index
2019	22949	110	1.4
2020	23706	94	0.04
2021	24567	97	3.1
2022	25432	85	1.0
2023	26300	80	5.9

Source: Federal Statistical Office of Germany

Model Results

**Figure 10 Forecast Difference Plot for the German Pet Industry****Figure 11 Forecast Fitting Plot for the German Pet Industry**

From the difference plot, the cat numbers peaked in 2021, with the difference value reaching a maximum. Starting in 2022, the difference value sharply dropped to negative, with a significant decline observed in 2023, indicating a marked decrease in the number of cats during this period.

For dogs, the difference value remained close to zero throughout the observed period, indicating relatively stable numbers.

From the fitting plot, the model fits the cat numbers well, although it struggles slightly during years of sharp changes (e.g., 2023). The model fits the dog numbers exceptionally well, accurately capturing their stable downward trend.

Table 8 Forecast Results for the German Pet Industry (in 10,000s)

Year	Cat Forecast	Dog Forecast
2024	1433.13	980.38
2025	1499.76	948.52
2026	1604.52	917.43

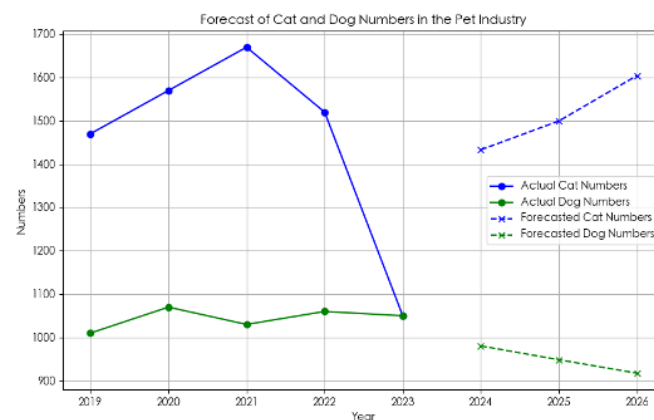


Figure 12 Forecast Plot for the German Pet Industry

The number of cats is forecasted to grow steadily from 14.33 million in 2024 to 16.05 million in 2026, indicating a recovery in market demand and a sustained growth trend. By 2026, the cat numbers are expected to approach the peak levels of 2021, suggesting a return to the growth trajectory.

The number of dogs is expected to decrease from 9.80 million in 2024 to 9.17 million in 2026, reflecting a gradual but consistent downward trend. Although the decline is small, the demand for dogs remains subdued, necessitating adjustments in market strategies.

V. The World's Pet Industry Forecast Based on Exponential Smoothing

5.1 Exponential Smoothing Model

We have actual data for China, the United States, France, and Germany from 2019 to 2023, as well as forecasted pet population values for 2024-2027. In 2023, the global total population of cats and dogs was 4220 (100,000s) and 5080 (100,000s), respectively. Based on the proportion of these four countries' data in 2023, combined with the exponential smoothing method, we can forecast the future trend and specific values of the global pet population.

Exponential smoothing is a commonly used time series forecasting method, especially suitable for data with trends and seasonality. In this study, we use the Holt-Winters exponential smoothing model to simultaneously capture the level, trend, and seasonality of the data.

For predicting the number of cats and dogs, this study uses double exponential smoothing, which is suitable for time series with trends but without seasonality:

$$S_t = \alpha Y_t + (1 - \alpha)(S_{t-1} + b_{t-1})$$

$$b_t = \beta(S_t - S_{t-1}) + (1 - \beta)b_{t-1}$$

5.2 Model Results

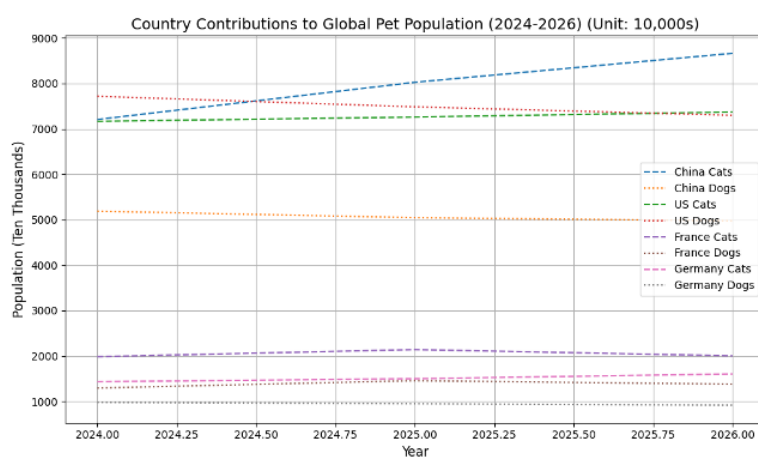


Figure 13 Visualization of forecasted values for the four countries

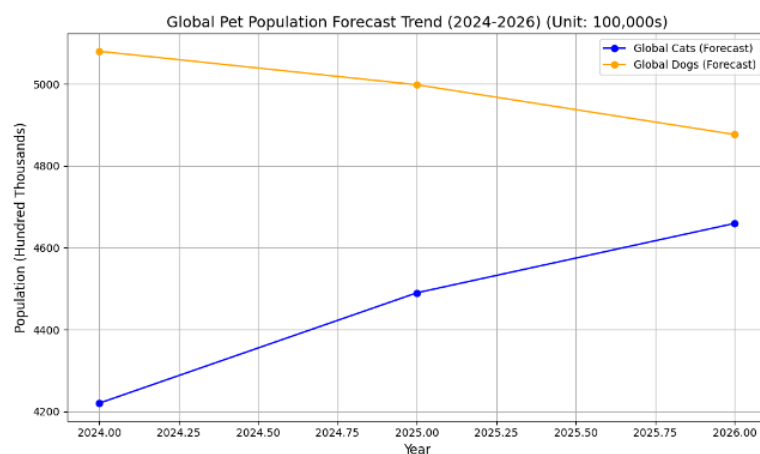


Figure 14 Global pet population forecast (2024-2026)

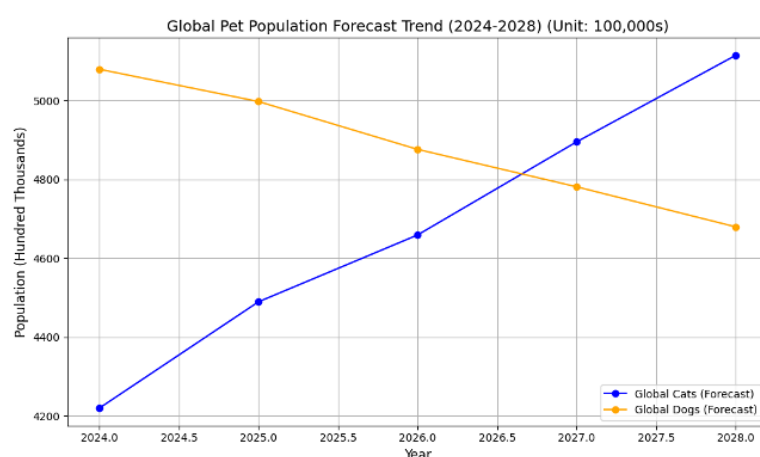


Figure 15 Global pet population forecast (2024-2028)

The global pet cat population is forecasted to increase steadily from 4220 (100,000s) in 2024 to 5115 (100,000s) in 2028. The growth rate accelerates year by year, indicating that cats are becoming increasingly popular.

The global pet dog population is projected to decline from 5080 (100,000s) in 2024 to 4680 (100,000s) in 2028. The rate of decline is relatively smooth, but it highlights a gradual decrease in dog ownership, which may be influenced by societal preferences or the costs of keeping dogs.

The global populations of cats and dogs are expected to converge around 2026, with the number of cats surpassing dogs in 2027 and beyond.

Table 9 Global Pet Industry Forecast Values (in 100,000s)

Year	Global_Cat	Global_Dog
2024	4220	5080
2025	4489.697742	4998.118643
2026	4659.287792	4876.726521
2027	4895.617250	4781.674926
2028	5115.261792	4680.038196

VI. Forecasting China's Pet Food Production and Export Using Holt Model

6.1 Current Analysis

From the analysis of production values, the total production value increased from 44.07 billion CNY in 2019 to 279.3 billion CNY in 2023, with an average annual growth rate of over 50%. The growth was particularly significant from 2020 to 2021, showing an increase of approximately 114%. In 2022, there was a slight decline, followed by a sharp rebound in 2023.

For export values, the total export value in 2019 was approximately 2.2 billion USD. There was a decline between 2019 and 2020; however, exports rose from 0.98 billion USD in 2020 to 3.96 billion USD in 2023, achieving an average annual growth rate of nearly 50%. This reflects the growing competitiveness of China's pet food products in the international market and the steady increase in demand.

6.2 The Holt Model

The Holt linear trend model is a time series forecasting method. It builds upon simple exponential smoothing by introducing a trend component, making it suitable for time series data with linear trends. The Holt model consists of three main components:

- **Level component (L_t):** Represents the current level of the time series.
- **Trend component (T_t):** Represents the rate of change in the time series.
- **Forecast (F_{t+h}):** Combines level and trend components to predict future values.

The equations are as follows:

- **Level update equation:**

$$L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$

- **Trend update equation:**

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

- **Forecast equation:**

$$F_{t+h} = L_t + hT_t$$

Given that pet food production and exports are influenced by multiple factors and exhibit significant fluctuations, a **90% confidence interval** was used to quantify the uncertainty of the forecasts. The following formulas were applied:

- **Residual variance:**

$$\sigma^2 = \frac{1}{n-1} \sum_{t=1}^n (Y_t - Y'_t)^2$$

- **Forecast variance:**

$$\text{Var}(F_{t+h}) = \sigma^2 h$$

- **Confidence interval:**

$$CI = F_{t+h} \pm t_{\alpha/2, n-2} \sqrt{\text{Var}(F_{t+h})}$$

6.3 Model Results

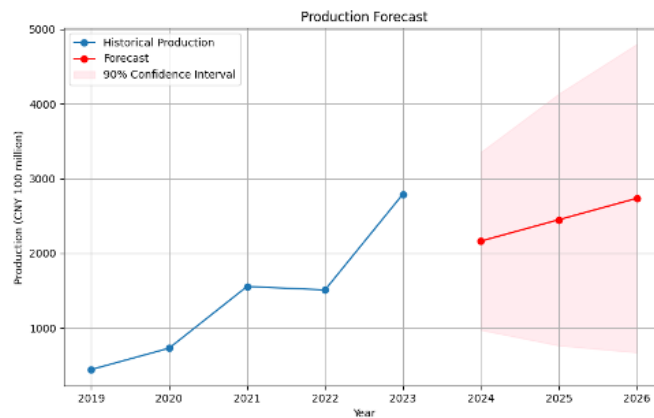


Figure 16 Forecast for China's pet food production values

Table 10 China’s Pet Food Production Value Forecast (CNY 100 million)

Year	Forecast	Lower CI	Upper CI
2024	2162.94	969.85	3356.04
2025	2449.54	762.25	4136.83
2026	2736.14	669.64	4802.64

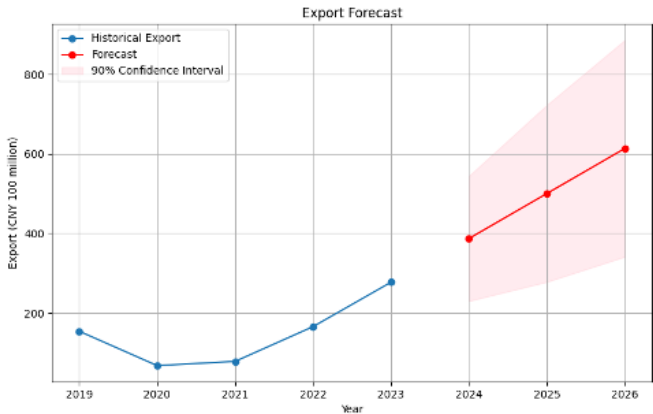


Figure 17 Forecast for China’s pet food export values

Table 11 China’s Pet Food Export Value Forecast (CNY 100 million)

Year	Forecast	Lower CI	Upper CI
2024	387.05	229.74	544.37
2025	500.34	277.86	722.82
2026	613.63	341.15	886.11

Based on the forecasts from the Holt linear trend model, China’s pet food production values are expected to grow steadily between 2024 and 2026. Compared with historical data, the growth rate is projected to slow down, and the confidence interval widens over time, indicating greater uncertainty in long-term forecasts. Meanwhile, export values are expected to grow strongly, reaching a confidence interval range of 341.15 to 886.11 billion CNY by 2026. This increasing uncertainty may be influenced by international trade policies, market competition, and macroeconomic factors.

VII. Econometric Model on Tariff Impact on China's Pet Food Industry

7.1 Econometric Model

Assuming that tariff levels remained constant during the period from 2019 to 2023, we further assume that the current export value serves as the baseline and tariffs increase from 5% to 50%. Based on international trade theory, the elasticity of exports to tariff changes is estimated using the following model:

According to international trade theory, the import demand for a product can be expressed as the export demand function:

$$Q_d = a - b \times P$$

The landed price of exported products includes tariffs, and the price can be represented as:

$$P = P_0(1 + t)$$

Substituting the tariff-inclusive price into the demand function gives:

$$Q_d = a - b \times P_0(1 + t)$$

To simplify the model, we assume the price elasticity of import demand is ω , defined as:

$$\omega = \frac{\% \Delta Q_d}{\% \Delta P}$$

For each tariff level, the demand is calculated as:

$$Q_{d_i} = Q_{d_0} \times \left(\frac{P_0(1 + t_0)}{P_0(1 + t_i)} \right)^{\omega}$$

7.2 Model Results

Table 12 Relationship Between Tariff Rates and Export Volume Changes

Tariff Rate	Export Volume Change Rate
5%	0%
10%	-6.80%
15%	-13.60%

Tariff Rate	Export Volume Change Rate
20%	-20.10%
25%	-26.30%
30%	-32.20%
35%	-37.90%
40%	-43.40%
45%	-48.60%
50%	-53.60%

From the econometric model, when the tariff rate is 5%, the export volume change rate is 0%, which serves as the baseline scenario where exports remain unchanged. As tariff rates increase, export volumes exhibit negative growth, and the rate of decline accelerates. The reduction in export volumes is not linear; instead, as tariffs increase by 5%, the magnitude of the export reduction grows incrementally.

7.3 Sustainable Development Strategies

1. **Market Diversification** Actively expand into emerging markets such as Southeast Asia, Latin America, Africa, and the Middle East to reduce dependence on the European and American markets. Strengthen marketing and distribution channels in markets with lower tariff barriers. Sales declines in high-tariff markets can be offset by penetrating new markets.
2. **Product Upgrades and Differentiation** Enhance quality control to meet international standards and develop organic, natural, and functional pet food products to cater to high-end market demand. High-quality and differentiated products make consumers less price-sensitive, mitigating the negative impact of tariffs.
3. **Localized Production and Overseas Investment** Establish production bases in target markets or third-party countries with lower tariffs. Collaborate with local businesses to leverage their channels and resources for quicker market entry. This approach can satisfy local consumer demand more efficiently while improving service quality.
4. **Exploring Domestic Market Potential** With the rapid growth of China's pet market, the domestic market holds significant potential. Develop products and services tailored to the preferences of domestic consumers. Growth in the domestic market can offset some of the export declines and reduce reliance on external

markets.

VIII. Further Discussion on the Model

8.1 1. Model Advantages

1. The use of disposable income, consumer confidence index, and consumer price index as exogenous variables in the ARIMAX model allows for a more comprehensive capture of the economic environment affecting the changes in cat and dog numbers.
2. The ADF test is employed to determine whether the time series is stationary. For non-stationary series, differencing is applied to address the basic assumptions of time series analysis.
3. Standardization of exogenous variables ensures comparability among variables in the model, avoiding the impact of scale differences on the results.
4. Exponential smoothing is used to model the time series of pet numbers in known countries, capturing trend changes and providing smoother predictions for future years.
5. Confidence intervals are calculated using the residual standard deviation and the t -distribution, providing a range of uncertainty for the predictions, which enhances the credibility and interpretability of the results.

8.2 2. Model Limitations

1. The data spans only five years, resulting in a small sample size. This may lead to poor model fit and unreliable prediction results.
2. The ARIMAX model used does not account for seasonal effects. If the numbers of cats and dogs in the pet industry exhibit seasonal fluctuations, this could reduce the accuracy of the model.
3. Global predictions heavily rely on the known countries' pet numbers and proportions. If the predictions for the known countries are inaccurate or the proportional assumptions fail, the global total predictions may become unreliable.
4. The confidence intervals are based on the cumulative residual standard deviation, which widens over time. This could reduce the practicality of long-term prediction results.

IX. References

- [1] Jiang Keyang, Analysis of Development Issues and Solutions in China's Pet Industry, Modern Animal Husbandry and Veterinary Medicine, 2017, 5: 55-58.
- [2] Meng Gang, The Current Status and Prospects of China's Pet Industry, Wenzhou Agricultural Science and Technology, 2010 (2): 8-10.
- [3] Peter Ď, Silvia P, ARIMA vs. ARIMAX–Which Approach is Better to Analyze and Forecast Macroeconomic Time Series, Proceedings of 30th International Conference Mathematical Methods in Economics, 2012, 2: 136-140.
- [4] Bao Yanke, Chen Ran, Zheng Hongjie, et al., A Combined Forecasting Model Based on Exponential Smoothing and Autoregression: Empirical Evidence, Journal of Computer Engineering & Applications, 2022, 58(14).
- [5] Chen Liang, Huang Peng, Reconsideration of China's Tariff Policy Under the Reconstruction of International Trade Rules, Scientific Development, 2022.
- [6] Creedy J, Tax Modelling, Taxation and Economic Behaviour, Edward Elgar Publishing, 2001: 133-164.

X. Appendix

Problem 1

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.stattools import adfuller
from sklearn.preprocessing import StandardScaler
from matplotlib import rcParams

rcParams['font.sans-serif'] = ['PingFang', 'STHeiti']
rcParams['axes.unicode_minus'] = False

data = {
```

```
'year': [2019, 2020, 2021, 2022, 2023],
'cat': [1470, 1570, 1670, 1520, 1050],
'dog': [1010, 1070, 1030, 1060, 1050],
'disposable_income': [22949, 23706, 24567, 25432, 26300],
'consumer_confidence_index': [110, 94, 97, 85, 80],
'cpi': [1.4, 0.04, 3.1, 1, 5.9]
}
df = pd.DataFrame(data)
df.set_index('year', inplace=True)

def adf_test(series):
    result = adfuller(series)
    print(f'ADF Statistic: {result[0]}')
    print(f'p-value: {result[1]}')
    if result[1] < 0.05:
        print('The series is stationary')
    else:
        print('The series is not stationary')

print("cat ADF:")
adf_test(df['cat'])
print("\n dog ADF:")
adf_test(df['dog'])

df['cat_diff'] = df['cat'].diff()
df['dog_diff'] = df['dog'].diff()

plt.figure(figsize=(10, 6))
plt.plot(df.index[1:], df['cat_diff'].dropna(), label='Difference in
    Cat Numbers', color='b')
plt.plot(df.index[1:], df['dog_diff'].dropna(), label='Difference in
    Dog Numbers', color='g')
plt.title('Differenced Data for Cat and Dog Numbers')
plt.xlabel('Year')
plt.ylabel('Differenced Values')
plt.legend()
plt.grid(True)
```

```
plt.show()

print("\n income ADF:")
adf_test(df['disposable_income'])
print("\n CCI ADF:")
adf_test(df['consumer_confidence_index'])
print("\n CPI ADF:")
adf_test(df['cpi'])

scaler = StandardScaler()
df[['disposable_income', 'consumer_confidence_index', 'cpi']] =
    scaler.fit_transform(df[['disposable_income',
        'consumer_confidence_index', 'cpi']])

def arimax_model(target, exog_vars, df):
    model = SARIMAX(df[target], exog=df[exog_vars], order=(1, 1, 1),
        seasonal_order=(0, 0, 0, 0))
    result = model.fit(dispatch=False)
    return result

cat_exog_vars = ['disposable_income', 'consumer_confidence_index',
    'cpi']
cat_model = arimax_model('cat', cat_exog_vars, df)

dog_exog_vars = ['disposable_income', 'consumer_confidence_index',
    'cpi']
dog_model = arimax_model('dog', dog_exog_vars, df)

plt.figure(figsize=(10, 6))
plt.plot(df.index, df['cat'], label='Actual Cat Numbers', marker='o',
    color='b')
plt.plot(df.index, cat_model.fittedvalues, label='Fitted Cat Numbers',
    color='b', linestyle='--')
plt.plot(df.index, df['dog'], label='Actual Dog Numbers', marker='o',
    color='g')
plt.plot(df.index, dog_model.fittedvalues, label='Fitted Dog Numbers',
    color='g', linestyle='--')
```

```
plt.title('Actual and Fitted Values of Cat and Dog Numbers')
plt.xlabel('Year')
plt.ylabel('Numbers')
plt.legend()
plt.grid(True)
plt.show()

future_years = [2024, 2025, 2026]
future_data = {
    'disposable_income': [27000, 28000, 29000],
    'consumer_confidence_index': [90, 90, 90],
    'cpi': [2.3, 2.3, 2.3]
}

future_df = pd.DataFrame(future_data)
future_df = scaler.transform(future_df)

cat_forecast = cat_model.predict(start=len(df), end=len(df) + 2,
    exog=future_df)
dog_forecast = dog_model.predict(start=len(df), end=len(df) + 2,
    exog=future_df)

forecast_df = pd.DataFrame({
    'Year': future_years,
    'Cat Forecast': cat_forecast,
    'Dog Forecast': dog_forecast
})

print(forecast_df)

plt.figure(figsize=(10, 6))
plt.plot(df.index, df['cat'], label='Actual Cat Numbers', marker='o',
    color='b')
plt.plot(df.index, df['dog'], label='Actual Dog Numbers', marker='o',
    color='g')
plt.plot(future_years, cat_forecast, label='Forecasted Cat Numbers',
    marker='x', color='b', linestyle='--')
```

```
plt.plot(future_years, dog_forecast, label='Forecasted Dog Numbers',
         marker='x', color='g', linestyle='--')
plt.title('Forecast of Cat and Dog Numbers in the Pet Industry')
plt.xlabel('Year')
plt.ylabel('Numbers')
plt.legend()
plt.grid(True)
plt.show()
```

Problem 2

```
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.holtwinters import ExponentialSmoothing

data = {
    "Year": [2024, 2025, 2026],
    "China_Cat": [7206.627652, 8025.770635, 8663.701947],
    "China_Dog": [5187.788642, 5045.032247, 4976.539855],
    "US_Cat": [7165.661027, 7260.577783, 7370.568071],
    "US_Dog": [7717.433848, 7485.2854, 7299.398827],
    "France_Cat": [1984.564509, 2140.820475, 2003.070404],
    "France_Dog": [1294.229224, 1456.315187, 1379.050852],
    "Germany_Cat": [1433.129579, 1499.76131, 1604.519656],
    "Germany_Dog": [980.377832, 948.522492, 917.427264],
}

df = pd.DataFrame(data)

df["Known_Cat"] = df[["China_Cat", "US_Cat", "France_Cat",
                     "Germany_Cat"]].sum(axis=1)
df["Known_Dog"] = df[["China_Dog", "US_Dog", "France_Dog",
                     "Germany_Dog"]].sum(axis=1)

global_2023_cat = 4220
global_2023_dog = 5080
```

```
global_2023_cat_wan = global_2023_cat * 10
global_2023_dog_wan = global_2023_dog * 10

known_cat_ratio = df.loc[0, "Known_Cat"] / global_2023_cat_wan
known_dog_ratio = df.loc[0, "Known_Dog"] / global_2023_dog_wan

def forecast_exponential_smoothing(series, forecast_periods=3,
    seasonal=None):

    model = ExponentialSmoothing(
        series,
        trend='add',
        seasonal=seasonal,
        seasonal_periods=1
    )
    model_fit = model.fit()
    forecast = model_fit.forecast(forecast_periods)
    return forecast

known_cat_series = df.set_index("Year")["Known_Cat"]
known_dog_series = df.set_index("Year")["Known_Dog"]

forecast_periods = 2

cat_forecast = forecast_exponential_smoothing(known_cat_series,
    forecast_periods=forecast_periods)

dog_forecast = forecast_exponential_smoothing(known_dog_series,
    forecast_periods=forecast_periods)

forecast_years = list(range(2024, 2026 + forecast_periods + 1)) #
2024-2028
forecast_df = pd.DataFrame({
    "Year": forecast_years,
    "Known_Cat": list(known_cat_series) + list(cat_forecast),
    "Known_Dog": list(known_dog_series) + list(dog_forecast)
```

```
})

known_cat_ratio_forecast = forecast_df.loc[0, "Known_Cat"] /
    global_2023_cat_wan
known_dog_ratio_forecast = forecast_df.loc[0, "Known_Dog"] /
    global_2023_dog_wan

forecast_df["Global_Cat"] = forecast_df["Known_Cat"] /
    known_cat_ratio_forecast / 10
forecast_df["Global_Dog"] = forecast_df["Known_Dog"] /
    known_dog_ratio_forecast / 10

print("Global Pet Population Forecast (2024-2028) :")
print(forecast_df[["Year", "Global_Cat", "Global_Dog"]])

plt.figure(figsize=(10, 6))
plt.plot(forecast_df["Year"], forecast_df["Global_Cat"], label="Global
    Cats (Forecast)", marker="o", color="blue")
plt.plot(forecast_df["Year"], forecast_df["Global_Dog"], label="Global
    Dogs (Forecast)", marker="o", color="orange")
plt.title("Global Pet Population Forecast Trend (2024-2028) (Unit:
    100,000s)", fontsize=14)
plt.xlabel("Year", fontsize=12)
plt.ylabel("Population (Hundred Thousands)", fontsize=12)
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()

plt.figure(figsize=(10, 6))
for country in ["China", "US", "France", "Germany"]:
    plt.plot(forecast_df["Year"],
        forecast_df[f"{country}_Cat"] if f"{country}_Cat" in
            forecast_df else [None] * len(forecast_df["Year"]),
        label=f"{country} Cats", linestyle="--")
    plt.plot(forecast_df["Year"],
```

```
        forecast_df[f"{country}_Dog"] if f"{country}_Dog" in
        forecast_df else [None] * len(forecast_df["Year"]),
        label=f"{country} Dogs", linestyle=":")
plt.title("Country Contributions to Global Pet Population (2024-2028)
        ", fontsize=14)
plt.xlabel("Year", fontsize=12)
plt.ylabel("Population (Ten Thousands)", fontsize=12)
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()

countries = ["China", "US", "France", "Germany"]

for year in forecast_df["Year"]:
    plt.figure(figsize=(12, 6))

    global_cat = forecast_df.loc[forecast_df["Year"] == year,
        "Global_Cat"].values[0]
    global_dog = forecast_df.loc[forecast_df["Year"] == year,
        "Global_Dog"].values[0]

    contributions_cat = [
        df.loc[0, "China_Cat"],
        df.loc[0, "US_Cat"],
        df.loc[0, "France_Cat"],
        df.loc[0, "Germany_Cat"],
    ]
    contributions_dog = [
        df.loc[0, "China_Dog"],
        df.loc[0, "US_Dog"],
        df.loc[0, "France_Dog"],
        df.loc[0, "Germany_Dog"],
    ]

    width = 0.35
    x = range(len(countries))
```

```

plt.bar(x, contributions_cat, width=width, color="blue", alpha=0.7,
        label="Known Countries Cats")
plt.axhline(global_cat * 10, color="red", linestyle="--",
            label="Global Total Cats ")

plt.bar([i + width for i in x], contributions_dog, width=width,
        color="orange", alpha=0.7,
        label="Known Countries Dogs")
plt.axhline(global_dog * 10, color="green", linestyle="--",
            label="Global Total Dogs")

plt.title(f"{year} Pet Population: Countries vs Global Total ",
        fontsize=14)
plt.xlabel("Country", fontsize=12)
plt.ylabel("Pet Population (Ten Thousands / Hundred Thousands)",
        fontsize=12)
plt.xticks([i + width / 2 for i in x], countries)
plt.legend()
plt.grid(axis='y')
plt.tight_layout()
plt.show()

```

Problem 3

```

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.holtwinters import Holt
from statsmodels.tools.eval_measures import rmse
from scipy.stats import t

years = np.array([2019, 2020, 2021, 2022, 2023])

production_values = np.array([440.7, 727.3, 1554, 1508, 2793])

```

```
export_values = np.array([154.1, 67.62, 78.08, 165.49, 277.20])

data = pd.DataFrame({
    'Year': years,
    'Production': production_values,
    'Export': export_values
})
data.set_index('Year', inplace=True)

plt.figure(figsize=(10, 6))
plt.plot(data.index, data['Production'], marker='o')
plt.title('China Pet Food Production (2019-2023)')
plt.xlabel('Year')
plt.ylabel('Production (CNY 100 million)')
plt.grid(True)
plt.show()

plt.figure(figsize=(10, 6))
plt.plot(data.index, data['Export'], marker='o', color='green')
plt.title('China Pet Food Export Value (2019-2023)')
plt.xlabel('Year')
plt.ylabel('Export Value (CNY 100 million)')
plt.grid(True)
plt.show()

def holt_forecast(series, forecast_periods, alpha=0.1):

    model = Holt(series).fit(optimized=True)

    print(f'\nModel parameters for {series.name}:')

    smoothing_level = model.params.get('smoothing_level', None)
    smoothing_trend = model.params.get('smoothing_trend', None)
    if smoothing_level is not None:
        print(f'Level (alpha): {smoothing_level:.4f}')
    else:
        print('Level (alpha): Not available')
```

```
if smoothing_trend is not None:
    print(f'Trend (beta): {smoothing_trend:.4f}')
else:
    print('Trend (beta): Not available')

forecast = model.forecast(steps=forecast_periods)
forecast_years = np.arange(series.index[-1] + 1, series.index[-1] +
    forecast_periods + 1)
forecast.index = forecast_years

residuals = series - model.fittedvalues
sigma = np.std(residuals, ddof=1)
df = len(series) - 2
t_value = t.ppf(1 - alpha / 2, df)

forecast_variance = sigma ** 2 * np.arange(1, len(forecast) + 1)
forecast_std = np.sqrt(forecast_variance)
conf_int_upper = forecast + t_value * forecast_std
conf_int_lower = forecast - t_value * forecast_std

forecast_df = pd.DataFrame({
    'Forecast': forecast,
    'Lower CI': conf_int_lower,
    'Upper CI': conf_int_upper
})

print(f'\nForecasted {series.name} with {int((1 - alpha) * 100)}%
    Confidence Intervals:')
print(forecast_df)

plt.figure(figsize=(10, 6))
plt.plot(series.index, series, marker='o', label=f'Historical
    {series.name}')
plt.plot(forecast_df.index, forecast_df['Forecast'], marker='o',
    color='red', label='Forecast')
plt.fill_between(forecast_df.index, forecast_df['Lower CI'],
    forecast_df['Upper CI'], color='pink', alpha=0.3,
```

```
        label=f'{int((1 - alpha) * 100)}% Confidence  
        Interval')  
plt.title(f'{series.name} Forecast')  
plt.xlabel('Year')  
plt.ylabel(f'{series.name} (CNY 100 million)')  
plt.legend()  
plt.grid(True)  
plt.show()  
  
return forecast_df  
  
production_forecast = holt_forecast(data['Production'],  
    forecast_periods=3)  
  
export_forecast = holt_forecast(data['Export'], forecast_periods=3)
```